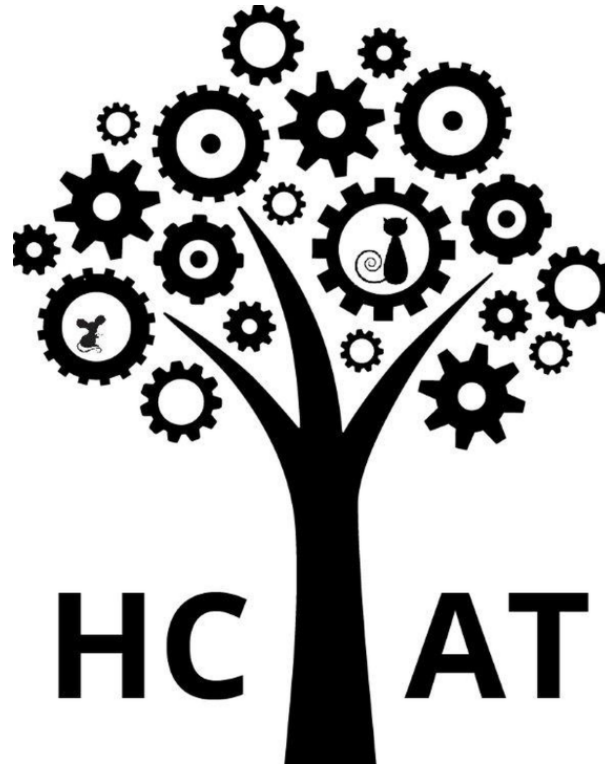


Year 1	Year 2
Year 3	Year 4
Year 5	Year 6



Mathematics Curriculum

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Aims

The HCAT Maths Curriculum aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

Information and communication technology (ICT)

Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of Key Stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure. In both primary and secondary schools, teachers should use their judgement about when ICT tools should be used.

Spoken language

The National Curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.

The programmes of study for mathematics are set out year-by-year for Key Stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study. In addition, schools can introduce key stage content during an earlier key stage, if appropriate.

Attainment targets

By the end of each key stage, pupils are expected to know, apply and understand the matters, skills and processes specified in the relevant programme of study.



Maths Skills and Knowledge in EYFS

Statutory Education Programme: Developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be able to count confidently, develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers.

Skills and knowledge? (Fluid across FS1 FS2)		What this looks like in provision/adult interactions?	Transition to KS1	Characteristics of Effective Learning
Number	Numerical patterns			
- Combine objects like stacking blocks and cups. Put objects inside others and take them out again. - Take part in finger rhymes with numbers - React to changes of amount in a group of up to three items - Compare amounts saying lots more or same (0-3).	- Counting- like behaviour, such as making sounds, pointing, or saying some numbers in sequence. - Count in everyday contexts, sometimes skipping numbers (0-3).	- Support children to match their counting words with objects. - Counting opportunities wherever possible e.g. when lining up to wash hands, counting children on the carpet - Counting in everyday contexts. - Provide patterned material – gingham, polka dots, stripes etc. – and small objects to arrange in patterns. Use words like ‘repeated’ and ‘the same’ over and over.	- Recognises some numerals of personal significance. - Recognises numerals 1-5. - Count up to 3 objects saying one number name for each item. - Count actions or objects which cannot be moved. - Join in with rote counting beyond 10.	Children in EYFS learn at different rates and abilities through: • Playing and exploring – children investigate and experience things, and ‘have a go’.
- Fast recognition of up to 3 objects. Without having to count them individually (subitising). - Say one number name for each item in order 1,2,3,4,5. - Know that the last number reached when counting a small set of objects tells you how many there are in total (cardinal principle). - Show fingers numbers up to 5. - Experiment with their symbols and marks as well as numbers (3-4).	- Recites numbers past 5 - Link numerals and amounts e.g. showing the right number of objects to match the numeral, up to 5. - Compare quantities using language: more than, fewer than. - Solve real world mathematical problems with numbers up to 5 (3-4).	- Count things and then repeat the last number. For example: “1, 2, 3 cars”. Point out the number of things whenever possible; so, rather than just ‘chairs’, ‘apples’ or ‘children’, say ‘two chairs’, ‘three apples’ etc. - Using accelerated learning cycle to give context to all maths learning to make this familiar for children and within their interests. - Opportunities for counting throughout the day and using vocabulary such as one more one less. - Resources that will provide opportunities for this within provision and using OLP for high quality adult interactions.	- Count objects to 10 and begin to count beyond 10. - Count out up to six objects from a larger group. - Count an irregular arrangement of up to ten objects. - Estimates how many objects they can see and checks by counting them. - Place numbers in order.	• Active learning – children concentrate and keep on trying if they encounter difficulties, and enjoy achievements.
- Count objects, actions, and sounds. - Subitise. - Link the number symbol with its cardinal number value. - Compare numbers. - Explore the composition of numbers to 10. (4-5).	- Automatically recall number bonds for numbers 0-5 some to 10. - Continue, copy, and create repeating patterns. - Understand the one more than/one less than relationships between consecutive numbers. - Count beyond 10 (4-5).	- Show small quantities in familiar patterns (for example, dice) and random arrangements. Play games which involve quickly revealing and hiding numbers of objects. - Discuss the different ways children might record quantities (for example, scores in games), such as tallies, dots and using numeral cards. - Encouraging children solving real life problems within games e.g. how many more do I need to score to make it a draw? - Emphasise the parts within the whole: “There were 8 eggs in the incubator. Two have hatched and 6 have not yet hatched.” - Plan games which involve partitioning and recombining sets.	- Respond appropriately to key vocabulary and questions e.g. how many. - Solve simple problems encountered in play. - Use original numbers when describing the position of objects people or events. - Find the total number of items in two groups by counting all of them. - Say the number that is one more than a given number. - Begin to use vocabulary in adding and subtracting. - Recognise differences in quantities.	• Creating and thinking critically – children have and develop their own ideas, make links between ideas, and develop strategies for doing things

Early Learning Goals

- Have a deep understanding of number to 10, including the composition of each number (number)
- Subitise (recognise quantities without counting) up to 5 (number)
- Automatically recall (without reference to rhymes, counting or other aides) number bonds up to 5 and some number bonds to 10 including double faces (number)
- Verbally count beyond 20, recognising the pattern of the counting system (numerical patterns)
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity (numerical patterns)
- Explore and represent patterns within numbers up to 10, including events and odds, double facts and how quantities can be distributed equally (numerical patterns)

Key Stage 1

Pupils should be taught to work confidently with whole numbers, counting and place value. They should be able to work with numerals, understand mathematical language, and use the four operations in a practical way such as measuring and counting concrete objects. Pupils should also be able to recognize, draw and compare different shapes, name them correctly, and compare different quantities such as length, mass, capacity, time and money. By the end of Year 2 pupils must know the number bonds to 20 and be confident in using and understanding place value and be able to read and spell mathematical vocabulary

Year 1

Year 1 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
Number and place value	Number and place value
Pupils should be taught to: • count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number • count, read and write numbers to 100 in numerals, count in different multiples including ones, twos, fives and tens • given a number, identify one more and one less identify and represent numbers using concrete objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least • read and write numbers from 1 to 20 in digits and words.	Pupils should practise counting (1, 2, 3), ordering e.g. first, second, third), or to indicate a quantity (e.g. 3 apples, 2 centimetres), including solving simple concrete problems, until they are fluent. They should practice counting as reciting numbers and counting as enumerating objects, and counting in ones, twos, fives and tens from different multiples to develop their recognition of patterns in the number system (e.g. odd and even numbers). They connect these patterns with objects and with shapes, including through varied and frequent practice of increasingly complex questions. Pupils begin to recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by concrete objects and pictorial representations.

Addition and subtraction	Addition and subtraction
Pupils should be taught to: • read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs • represent and use number bonds and related subtraction facts within 20 • add and subtract one-digit and two-digit numbers to 20 ($9 + 9$, $18 - 9$), including zero • solve simple one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems.	Pupils should memorise and reason with number bonds to 10 and 20 in several forms (e.g. $9 + 7 = 16$; $16 - 7 = 9$; $7 = 16 - 9$). They should realise the effect of adding or subtracting zero. Pupils should combine and increase numbers, counting forwards and backwards. They should discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms put together, add, altogether, total, take away, difference between, more than and less than so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Multiplication and division	Multiplication and division
Pupils should be taught to: • solve simple one-step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Through grouping and sharing small quantities, pupils should begin to understand multiplication and division; doubling numbers and quantities, and finding simple fractions of objects, numbers and quantities. They should make connections between arrays, number patterns, and counting in twos, fives and tens.

Fractions	Fractions
Pupils should be taught to: - recognise, find and name a half as one of two equal parts of an object, shape or quantity - recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.	Pupils should be taught $\frac{1}{2}$ and $\frac{1}{4}$ as operators on discrete and continuous quantities by solving problems using shapes, objects and quantities. For example, they could recognise and find half a length, quantity, set of objects or shape. Pupils connect halves and quarters to the equal sharing and grouping of sets of objects and to measures, as well as recognising and combining halves and quarters as parts of a whole.

Measures	Measures
Pupils should be taught to: - compare, describe and solve practical problems for: - lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half) - mass or weight (e.g. heavy/light, heavier than, lighter than) - capacity/volume (full/empty, more than, less than, quarter) - time (quicker, slower, earlier, later) - measure and begin to record the following: - lengths and heights - mass/weight - capacity and volume	The terms mass and weight, volume and capacity are used interchangeably at this stage. Pupils should move from using and comparing different types of quantities and measures using non-standard units, including discrete (e.g. counting) and continuous (e.g. liquid) measures, to using manageable common standard units. They should understand the difference between non-standard and standard units.

○ time (hours, minutes, seconds) • recognise and know the value of different denominations of coins and notes • sequence events in chronological order using language such as: before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening • recognise and use language relating to dates, including days of the week, weeks, months and years • tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.	In order to become familiar with standard measures, pupils begin to use measuring tools such as a ruler, weighing scales and containers. Pupils should use the language of time, including telling the time throughout the day, first using o'clock and then half past.
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Geometry: properties of shapes	Geometry: properties of shapes
Pupils should be taught to: • recognise and name common 2-D and 3-D shapes, including: • 2-D shapes (e.g. rectangles (including squares), circles and triangles) • 3-D shapes (e.g. cuboids (including cubes), pyramids and spheres).	Pupils should handle common 2-D and 3-D shapes, naming these and related everyday objects fluently. They should recognise these shapes in different orientations and sizes, and know that rectangles, triangles, cuboids and pyramids can be different shapes.

Geometry: position, direction, motion	Geometry: position, direction, motion
Pupils should be taught to:	Pupils should create, copy, describe and reorganise patterns.

• order and arrange combinations of objects and shapes in patterns • describe position, directions and movements, including half, quarter and three-quarter turns.	They should use the language of position, direction and motion, including: left and right, top, middle and bottom, on top of, in front of, above, between, around, near, close and far, up and down, forwards and backwards, inside and outside. Pupils should make turns to show they understand half, quarter and three-quarter turns and routinely make these turns in a clockwise direction.
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Year 2

Year 2 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
Number and place value	Number and place value
Pupils should be taught to: • count in steps of 2, 3, and 5 from 0, and count in tens from any number, forward or backward • recognise the place value of each digit in a two-digit number (tens, ones) • identify, represent and estimate numbers using different representations, including the number line • compare and order numbers from 0 up to 100; use $<$, $>$ and $=$ signs • read and write numbers to at least 100 in numerals and in words use place value and number facts to solve problems.	Using materials and a range of representations, pupils should practise counting, reading, writing and comparing numbers to at least 100 and solving a variety of related problems to develop fluency. They should count in multiples of three to support their later understanding of a third. As they become more confident with numbers up to 100, pupils should be introduced to larger numbers to develop further their recognition of patterns within the number system and represent them in different ways, including spatial representations. Pupils should partition numbers in different ways (e.g. $23 = 20 + 3$ and $23 = 10 + 13$) to support subtraction. They become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that

	emphasise the value of each digit in two-digit numbers. They begin to understand zero as a place holder.
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Addition and subtraction	Addition and subtraction
Pupils should be taught to: • solve simple one-step problems with addition and subtraction: ○ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ○ applying their increasing knowledge of mental and written methods • recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: ○ a two-digit number and ones ○ a two-digit number and tens ○ two two-digit numbers ○ adding three one-digit numbers • show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot	Pupils should extend their understanding of the language of addition and subtraction to include sum and difference. Pupils should practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using $3 + 7 = 10$, $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $30 + 70 = 100$, $100 - 70 = 30$ and $70 = 100 - 30$. They should check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (e.g. $5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5$). Recording addition and subtraction in columns supports place value and prepares for efficient written methods with larger numbers.

recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.	
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Multiplication and division	Multiplication and division
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Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equals (=) signs
- recognise and use the inverse relationship between multiplication and division in calculations
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

solve one-step problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Pupils should use a variety of language to describe multiplication and division. They are taught multiplication and division with larger numbers through equal grouping and sharing out quantities, relating multiplication tables to arrays and repeated addition and finding more complex fractions of objects, numbers and quantities.

Pupils should be introduced to the multiplication tables. They should practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10-multiplication table to place value, and the 5-multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

Pupils should work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, relating these to fractions and measures (e.g. $40 \div 2 = 20$, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (e.g. $4 \times 5 = 20$ and $20 \div 5 = 4$).

Fractions	Fractions
Pupils should be taught to: - recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity - write simple fractions e.g. $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of two quarters and one half.	Pupils should use additional fractions as operators on discrete and continuous quantities by solving problems using shapes, objects and quantities. They connect unit fractions to equal sharing and grouping, to numbers when they can be calculated, and to measures, finding fractions of lengths, quantity, a set of objects or shapes. They meet $\frac{3}{4}$ as the first example of a non-unit fraction. Pupils should count in fractions up to 10, starting from any number and using the $\frac{1}{2}$ and $\frac{2}{4}$ equivalence on the number line (e.g. $1\frac{1}{4}$, $1\frac{2}{4}$, (or $1\frac{1}{2}$), $1\frac{3}{4}$, 2). This reinforces the concept of fractions as numbers and that they can add up to more than one.

Measures	Measures
Pupils should be taught to: - choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature ($^{\circ}\text{C}$); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels - compare and order lengths, mass, volume/capacity and record the results using $>$, $<$ and $=$ - read relevant scales to the nearest numbered unit - recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value and match	Pupils should use standard units of measurement with increasing accuracy, using their knowledge of the number system. They should use the appropriate language and record using standard abbreviations. They should become fluent in telling the time on analogue clocks and recording it. Pupils should also become fluent in counting and recognising coins. They should use the symbols £ and p accurately and say the amounts of money confidently.

different combinations of coins to equal the same amounts of money; add and subtract money of the same unit, including giving change • solve simple problems in a practical context involving addition and subtraction of money • compare and sequence intervals of time tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times.	
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Geometry: properties of shapes	Geometry: properties of shapes
Pupils should be taught to: • identify and describe the properties of 2-D shapes, including the number of sides and symmetry in a vertical line • identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces • identify 2-D shapes on the surface of 3-D shapes, for example a circle on a cylinder and a triangle on a pyramid • compare and sort common 2-D and 3-D shapes and everyday objects.	Pupils should handle and name a wider variety of common 2-D and 3-D shapes including: quadrilaterals and cuboids, prisms, cones and polygons, and identify the properties of each shape (e.g. number of sides, number of faces). Pupils identify, compare and sort shapes on the basis of their properties and use vocabulary precisely, such as sides, edges, vertices and faces. Pupils should read and write names for shapes that are appropriate for their word reading and spelling. Pupils should draw lines and shapes using a straight edge.

Geometry: position, direction, motion	Geometry: position, direction, motion
Pupils should be taught to: • order and arrange combinations of mathematical objects in patterns • use mathematical vocabulary to describe position, direction and movement, including distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise), and movement in a straight line.	Pupils should work with patterns of shapes, including those in different orientations. Pupils should use the concept and language of angles to describe ‘turn’ by applying rotations, including in practical contexts (e.g. pupils themselves moving in turns, giving instructions to other pupils to do so, and programming robots using instructions given in right angles).

Data	Data
Pupils should be taught to: • interpret and construct simple pictograms, tally charts, block diagrams and simple tables • ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity • ask and answer questions about totalling and compare categorical data.	At this stage, pupils’ recording, and interpretation become more sophisticated as they collate, organise and compare information (e.g. using many-to-one correspondence in pictograms and using simple ratios 2, 5, 10).

Lower Key Stage 2 – Years 3-4

In lower Key Stage 2 teaching should build on the work of Key Stage 1 and pupils must be increasingly fluent with whole numbers and the four operations, including number facts and place value. Teachers should ensure that pupils can work both mentally and on paper with whole numbers. Problem solving using simple fractions and decimal place value should be developed and used with confidence by pupils. Mathematical reasoning should be developed so that pupils can recognize shapes and their properties and describe the relationships between them. Teaching should focus on ensuring pupils can measure with accuracy.

By the end of Year 4 pupils should have learnt their tables up to and including 12.

Year 3

Year 3 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
Number and place value	Number and place value
Pupils should be taught to: • count from 0 in multiples of 4, 8, 50 and 100; finding 10 or 100 more or less than a given number • recognise the place value of each digit in a three-digit number (hundreds, tens, ones) • compare and order numbers up to 1000 • identify, represent and estimate numbers using different representations • read and write numbers to at least 1000 in numerals and in words • solve number problems and practical problems involving these ideas.	Pupils should work with larger numbers, applying partitioning related to place value using varied and increasingly complex problems, building on work in Year 2 (e.g. $46 = 40$ and 6, $46 = 30$ and 16). Using a variety of representations, including those related to measure, pupils should continue to count in ones, tens and hundreds, so that they become fluent in the order and place value of numbers to 1000.

Addition and subtraction	Addition and subtraction
Pupils should be taught to: • add and subtract numbers mentally, including:	Pupils should practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.

○ a three-digit number and ones ○ a three-digit number and tens ○ a three-digit number and hundreds • add and subtract numbers with up to three digits, using the efficient written methods of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	Pupils should use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent.
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Multiplication and division	Multiplication and division
Pupils should be taught to: • recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers of times one-digit numbers, using mental and progressing to efficient written methods solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects.	Pupils should continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables Pupils should develop efficient mental methods, for example, using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$). Pupils should develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the efficient written methods of short multiplication and division.

	Pupils should solve simple problems in contexts, deciding which of the four operations to use and why, including measuring and scaling contexts, and correspondence problems in which m objects are connected to n objects (e.g. 3 hats and 4 coats, how many different outfits; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
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Fractions	Fractions
Pupils should be taught to: - count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 - recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators - recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators - recognise and show, using diagrams, equivalent fractions with small denominators - add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$) - compare and order unit fractions with the same denominator - solve problems that involve all of the above.	Pupils should connect tenths to place value and decimal measures, not restricted to decimals between 0 and 1 inclusive and to division by 10. They should begin to understand unit and non-unit fractions as numbers on the number line, and deduce relations between them, such as size and equivalence. They should go beyond the [0, 1] interval, and $\frac{1}{4} + \frac{3}{4} = 1$ for example, relating this to measure. Pupils should understand the relation between unit fractions as operators and division by integers. They should continue to recognise fractions in the context of parts of a whole, numbers, measurements, a shape, or unit fractions as a division of a quantity. Pupils should practise adding and subtracting fractions with the same denominator through a variety of increasingly complex problems to improve fluency.

Measures	Measures
Pupils should be taught to: • measure, compare, add and subtract lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml) • measure the perimeter of simple 2-D shapes • add and subtract amounts of money to give change, using both £ and p in practical contexts • tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks • estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as a.m./p.m., morning, afternoon, noon and midnight • know the number of seconds in a minute and the number of days in each month, year and leap year compare durations of events, for example to calculate the time taken by particular events or tasks.	Pupils should continue to measure using the appropriate tools and units, progressing to using a wider range of measures, including comparing and using mixed units (e.g. 1 kg and 200g) and simple equivalents of mixed units (e.g. 5m = 500cm). The comparison of measures should also include simple scaling (e.g. a given quantity or measure is twice as long or five times as high) and connect this to multiplication. Pupils should continue to become fluent in recognising the value of coins, by adding and subtracting amounts, including mixed units, and giving change using manageable amounts. They should record £ and p separately. The decimal recording of money is introduced formally in Year 4. Pupils should use both analogue and digital 12-hour clocks and record their times. In this way they become fluent in and prepared for using digital 24-hour clocks in Year 4.

Geometry: properties of shapes	Geometry: properties of shapes
Pupils should be taught to: - draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations; and describe them with increasing accuracy - recognise angles as a property of shape and associate angles with turning - identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle - identify horizontal, vertical, perpendicular and parallel lines in relation to other lines.	Pupils' knowledge of the properties of shapes is extended at this stage to symmetrical and non-symmetrical polygons and polyhedra. Pupils extend their use of the properties of shapes. They should be able to describe the properties of 2-D and 3-D shapes using accurate language, including lengths of lines and acute and obtuse for angles greater or lesser than a right angle. Pupils should draw and measure straight lines in centimetres.

Data	Data
Pupils should be taught to: - interpret and present data using bar charts, pictograms and tables - solve one-step and two-step questions such as 'How many more?' and 'How many fewer?' using information presented in scaled bar charts and pictograms and tables.	Pupils should understand and use simple scales (e.g. 2, 5, 10 units per cm) in pictograms and bar charts with increasing accuracy. They should continue to interpret data presented in many contexts.

Year 4

Year 4 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
Number and place value	Number and place value
Pupils should be taught to • count in multiples of 6, 7, 9, 25 and 1000 • find 1000 more or less than a given number • count backwards through zero to include negative numbers • recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) • order and compare numbers beyond 1000 • identify, represent and estimate numbers using different representations • round any number to the nearest 10, 100 or 1000 • solve number and practical problems that involve all of the above and with increasingly large positive numbers • read Roman numerals to 100 (I to C) and understand how, over time, the numeral system changed to include the concept of zero and place value.	Using a variety of representations, including measures, pupils should become fluent in the order and place value of numbers beyond 1000, including counting in tens and hundreds, and maintaining fluency in other multiples through varied and frequent practice. They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far. Roman numerals should be put in their historical context, so pupils understand that there have been different ways to write whole numbers and that the important concepts of zero and place value were introduced over a period of time.

Addition and subtraction	Addition and subtraction
Pupils should be taught to • add and subtract numbers with up to 4 digits using the efficient written methods of columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.	Pupils should continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency.

Multiplication and division	Multiplication and division
Pupils should be taught to: • recall multiplication and division facts for multiplication tables up to 12×12 • use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout	Pupils should continue to practise recalling and using multiplication tables and related division facts to aid fluency. Pupils should practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ into $600 \div 3 = 200$, to become fluent. Pupils should practise to become fluent in the efficient written method of short multiplication for multiplying using multi-digit numbers, and short division with exact answers when dividing by a one-digit number. Pupils should write statements about the equality of expressions (e.g. use the distributive law $39 \times 7 = 30 \times 7 + 9 \times 7$ and associative law $(2 \times 3) \times 4 = 2 \times (3 \times 4)$).

solve problems involving multiplying and adding, including using the distributive law and harder multiplication problems such as which n objects are connected to m objects.

Pupils should solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Fractions

Pupils should be taught to:

- count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- identify, name and write equivalent fractions of a given fraction, including tenths and hundredths
- add and subtract fractions with the same denominator.

Fractions

Pupils should connect hundredths to tenths and place value and decimal measure. They should extend the use of the number line to connect fractions, numbers and measures.

Pupils should understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths.

Pupils should associate fractions of a length, of a shape and as a representation of one whole or set of quantities. Pupils should use factors and multiples to recognise equivalent fractions and simplify where appropriate (e.g. $\frac{6}{9} = \frac{2}{3}$ or $\frac{1}{4} = \frac{2}{8}$).

Pupils should continue practice in adding and subtracting fractions with the same denominator, to become fluent through a variety of increasingly complex problems beyond one whole. They should practise counting using simple fractions and decimal fractions, both forwards and backwards.

Decimals and fractions	Decimals and fractions
Pupils should be taught to: - recognise and write decimal equivalents of any number of tenths or hundredths - recognise and write decimal equivalents to $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{4}$ - find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths - round decimals with one decimal place to the nearest whole number - compare numbers with the same number of decimal places up to two decimal places - solve simple measure and money problems involving fractions and decimals to two decimal places.	Pupils should be taught throughout that decimals and fractions are different ways of expressing numbers. Pupils' understanding of the number system and decimal place value is extended at this stage to tenths and then hundredths. This includes relating the decimal notation to division of whole numbers by 10 and later 100. Pupils should learn decimal notation and the language associated with it, including in the context of measurements. They make comparisons and order decimal amounts and quantities that are expressed to the same number of decimal places. They should be able to represent numbers with one or two decimal places in multiple ways, such as on number lines.

Measures	Measures
Pupils should be taught to: - convert between different units of measure (e.g. kilometre to metre, hour to minute) - measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres - find the area of rectilinear shapes by counting	Pupils should use multiplication and their knowledge of place value to convert from larger to smaller units. They should relate area to arrays and multiplication.

- estimate, compare and calculate different measures, including money in pounds and pence - read, write and convert time between analogue and digital 12 and 24-hour clocks solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.	Pupils should build on their understanding of decimal notation to record measures.
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Geometry: properties of shapes	Geometry: properties of shapes
Pupils should be taught to: - compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes - identify acute and obtuse angles and compare and order angles up to two right angles by size - identify lines of symmetry in 2-D shapes presented in different orientations - complete a simple symmetric figure with respect to a specific line of symmetry.	Pupils should continue to classify shapes using geometrical properties, extending to classifying different triangles (e.g. isosceles, equilateral, scalene) and quadrilaterals (e.g. parallelogram, rhombus, trapezium). Pupils should compare and order angles in preparation for using a protractor and compare lengths and angles to decide if a polygon is regular or irregular. Pupils should draw symmetric patterns using a variety of media to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a variety of diagrams.

Geometry: position, direction, motion	Geometry: position, direction, motion
Pupils should be taught to: - describe positions on a 2-D grid as coordinates in the first quadrant - describe movements between positions as translations of a given unit to the left/right and up/down - plot specified points and draw sides to complete a given polygon.	Pupils should draw a pair of axes in one quadrant, with equal scales and integer labels. They should read, write and use pairs of coordinates (2, 5), including using coordinate-plotting ICT tools.

Data	Data
Pupils should be taught to: - interpret and present discrete data using bar charts and continuous data using line graphs - solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and simple line graphs.	Pupils should understand and use a greater range of scales in their representations. Pupils should begin to relate the graphical representation of data to recording change over time.

Upper Key Stage 2 – Years 5-6

In upper Key Stage 2 teaching should focus on ensuring pupils work confidently with larger numbers and consolidate their understanding of the number system and place value. They should make connections between multiplication and division with fractions, decimals, percentages and ratios.

Problem solving should widen to include more complex number work and arithmetic using written and mental calculations. Pupils are introduced to algebra as a means of problem solving. Geometry and measure should be taught so that pupils can extend their use of number, and shapes must be classified using accurate vocabulary.

By the end of Year 6 pupils should be fluent in all four operations including long multiplication and division, fractions, decimals and percent

Year 5

Year 5 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
Number and place value	Number and place value
Pupils should be taught to: • read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit • count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 • interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers through zero • round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 • solve number problems and practical problems that involve all of the above • read Roman numerals to 1000 (M) and recognise years written in Roman numerals.	Pupils should identify the place value in large whole numbers. They should continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions that they have met so far. They should recognise and describe linear number sequences, including those involving fractions and decimals, and find the term-to-term rule.

Addition and subtraction	Addition and subtraction
Pupils should be taught to: • add and subtract whole numbers with more than 4 digits, including using efficient written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	Pupils should practise using the efficient written methods of columnar addition and subtraction with increasingly large numbers to aid fluency. They should practise mental calculations with increasingly large numbers to aid fluency (e.g. $12\ 462 - 2\ 300 = 10\ 162$).

Multiplication and division	Multiplication and division
Pupils should be taught to: • identify multiples and factors, including finding all factor pairs • solve problems involving multiplication and division where larger numbers are used by decomposing them into their factors • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers	Pupils should practise and extend their use of the efficient written methods of short multiplication and short division. They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations. They should use and understand the terms factor, multiple and prime, square and cube numbers. Pupils should interpret non-integer answers to division by expressing results in different ways according to the context, including with

- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using an efficient written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the efficient written method of short division and interpret remainders appropriately for the context

recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)

- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equal's sign

solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24$ r $2 = 24\frac{1}{2} = 24.5 \approx 25$).

Pupils use multiplication and division as inverses to support the introduction of ratio in Year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.

Fractions	Fractions
Pupils should be taught to: • compare and order fractions whose denominators are all multiples of the same number • recognise mixed numbers and improper fractions and convert from one form to the other • add and subtract fractions with the same denominator and related fractions; write mathematical statements >1 as a mixed number (e.g. $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$) • multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.	Pupils should connect equivalent fractions >1 that simplify to integers with division and fractions >1 to division with remainders, using the number line and other models, and hence move from these to improper and mixed fractions. Pupils should connect multiplication by a fraction to using fractions as operators, and to division, building on work from previous years. This relates to scaling by simple fractions. They should extend their knowledge of fractions to thousandths and connect to decimals and measures. Pupils continue to develop their understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities, writing remainders as a fraction. Pupils should practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They should extend their understanding of adding and subtracting fractions to calculations that exceed 1 as a mixed number. Pupils should read and write proper fractions and mixed numbers accurately and continue to practise counting forwards and backwards with mixed fractions.

Decimals and fractions	Decimals and fractions
Pupils should be taught to: • read and write decimal numbers as fractions (e.g. $0.71 = \frac{71}{100}$) • recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents • round decimals with two decimal places to the nearest whole number and to one decimal place • read, write, order and compare numbers with up to three decimal places • solve problems involving number up to three decimal places.	Pupils extend counting from Year 4, using decimals and fractions including bridging zero, for example on a number line. They should add and subtract decimals including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (e.g. $0.83 + 0.17 = 1$). They should mentally add and subtract tenths, and one-digit whole numbers and tenths. Pupils should say, read and write decimal fractions and related tenths, hundredths and thousandths accurately and be confident in checking the reasonableness of their answers to problems. Pupils should go beyond the measurement and money models of decimals, for example by solving puzzles involving decimals.

Percentages, decimals and fractions	Percentages, decimals and fractions
Pupils should be taught to: - recognise the per cent symbol (%) and understand that per cent relates to “number of parts per hundred”, and write percentages as a fraction with denominator hundred, and as a decimal fraction - solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those with a denominator of a multiple of 10 or 25.	Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing numbers. Pupils should make connections between percentages, fractions and decimals (e.g. 100% represents a whole quantity and 1% is $\frac{1}{100}$, 50% is $\frac{50}{100}$, 25% is $\frac{25}{100}$) and relate this to finding ‘fractions of’. They recognise that percentages are proportions of quantities as well as operators on quantities.

Measures	Measures
Pupils should be taught to: - convert between different units of measure (e.g. kilometre and metre; metre and centimetre; centimetre and millimetre; kilogram and gram; litre and millilitre) - understand and use basic equivalences between metric and common imperial units and express them in approximate terms - measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres - calculate and compare the area of squares and rectangles including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes	Pupils should use their knowledge of place value and multiplication and division to convert between standard units. Pupils should calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths. Missing number questions such as these are the beginning of algebraic understanding. They should also calculate the area of scale drawings using given measurements. Pupils should use all four operations in problems involving time and money, including conversions (e.g. days to weeks, leaving the answer as weeks and days).

- recognise and estimate volume (e.g. using 1 cm³ blocks to build cubes and cuboids) and capacity (e.g. using water) - solve problems involving converting between units of time solve problems involving addition and subtraction of units of measure (e.g. length, mass, volume, money) using decimal notation.	
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Geometry: properties of shapes	Geometry: properties of shapes
Pupils should be taught to: - identify 3-D shapes, including cubes and cuboids, from 2-D representations - know angles are measured in degrees; estimate and measure them and draw a given angle, writing its size in degrees (°) - identify: - multiples of 90° - angles at a point on a straight line and $\frac{1}{2}$ a turn (total 180°) - angles at a point and one whole turn (total 360°) - reflex angles, and - compare different angles - draw shapes using given dimensions and angles - state and use the properties of a rectangle (including squares) to deduce related facts	Pupils should become accurate in drawing lines with a ruler to the nearest millimetre, and measuring with a protractor. They use conventional markings for parallel lines and right angles. Pupils should use the term diagonal and make conjectures about the angles formed by diagonals and sides, and other properties of quadrilaterals, for example using dynamic geometry ICT tools. Pupils should use angle sum facts and other properties to make deductions about missing angles and relate these to missing number problems.

distinguish between regular and irregular polygons based on reasoning about equal sides and angles.	
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Geometry: position, direction, motion	Geometry: position, direction, motion
Pupils should be taught to: identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.	Pupils should recognise and use reflection and translation in a variety of diagrams, including continuing to use a 2-D grid and coordinates in the first quadrant. Reflection should be in lines that are parallel to the axes.

Data	Data
Pupils should be taught to: - solve comparison, sum and difference problems using information presented in line graphs - complete, read and interpret information in tables, including timetables.	Pupils should connect their work on coordinates and scales to their interpretation of time graphs using ICT tools, except where data are easily calculable. They should begin to decide which representations of data are most appropriate and why.

Year 6

Year 6 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
Number and place value	Number and place value
Pupils should be taught to: • read, write, order and compare numbers up to 10 000 000 and determine the value of each digit • round any whole number to a required degree of accuracy • use negative numbers in context, and calculate intervals across zero • solve number problems and practical problems that involve all of the above.	Pupils should use the whole number system, including saying, reading and writing numbers accurately.

Addition, subtraction, multiplication and division	Addition, subtraction, multiplication and division
Pupils should be taught to: • multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • perform mental calculations, including with mixed operations and large numbers • identify common factors, common multiples and prime numbers • use their knowledge of the order of operations to carry out calculations involving the four operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.	Pupils should practise addition, subtraction, multiplication and division for larger numbers, using the efficient written methods of columnar addition and subtraction, short and long multiplication, and short and long division. They should undertake mental calculations with increasingly large numbers and more complex calculations. Pupils should continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils should round answers to a specified degree of accuracy. Pupils explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$. Common factors can be related to finding equivalent fractions.

Fractions	Fractions
Pupils should be taught to: • use common factors to simplify fractions; use common multiples to express fractions in the same denomination • compare and order fractions, including fractions >1 • associate a fraction with division to calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $\frac{3}{8}$) • add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions • multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$) • divide proper fractions by whole numbers (e.g. $\frac{1}{3} \div 2 = \frac{1}{6}$).	Pupils should use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (e.g. if $\frac{1}{4}$ of a length is 36cm, then the whole length is $36 \times 4 = 144\text{cm}$). They should practise with simple fractions and decimal fraction equivalents to aid fluency, including listing equivalent fractions to identify fractions with common denominators. Denominators of given fractions should not exceed 12, with the exception of 100 and 1000. Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (e.g. $3 \div 8 = 0.375$). For simple fractions with recurring decimal equivalents, pupils should learn about rounding the decimal to three decimal places. Pupils should practise, use and understand the addition and subtraction of fractions with different denominators by identifying equivalent fractions with the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other (e.g. $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$) and progress to varied and increasingly complex problems. Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators, as numbers, and as equal parts of objects, for example as parts of a rectangle.

Decimals and fractions	Decimals and fractions
Pupils should be taught to: - identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places - multiply one-digit numbers with up to two decimal places by whole numbers - use written division methods in cases where the answer has up to two decimal places - solve problems which require answers to be rounded to specified degrees of accuracy.	Pupils should begin to multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers. Pupils multiply decimals by whole numbers, starting with the simplest cases, such as $0.4 \times 2 = 0.8$, and in practical contexts, such as measures and money. Pupils should also be introduced to the division of decimal numbers by one-digit whole numbers and, initially, in practical contexts involving measures and money. They should recognise division calculations as the inverse of multiplication. Pupils should also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations. This includes rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.

Percentages, decimals and fractions	Percentages, decimals and fractions
Pupils should be taught to: - solve problems involving the calculation of percentages of whole numbers or measures such as 15% of 360 and the use of percentages for comparison - recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.	Pupils should understand that calculating a percentage of a quantity is the same as calculating a fraction of a quantity.

Ratio and proportion	Ratio and proportion
Pupils should be taught to: • solve problems involving the relative sizes of two quantities, including similarity • solve problems involving unequal sharing and grouping.	Pupils should consolidate their understanding of ratio when comparing quantities, sizes and scale drawings by solving a variety of problems. They may use the notation $a:b$ to record their work. Pupils should recognise proportionality in contexts when the relations between quantities are in the same ratio (e.g. similar shapes, recipes).

Algebra	Algebra
Pupils should be taught to: • express missing number problems algebraically • use simple formulae expressed in words • generate and describe linear number sequences • find pairs of numbers that satisfy number sentences involving two unknowns.	Pupils should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand, such as: • missing numbers, lengths, coordinates and angles • formulae in mathematics and science • arithmetical rules (e.g. $a + b = b + a$) • generalisations of number patterns • number puzzles (e.g. what two numbers can add up to).

Measures	Measures
Pupils should be taught to: • solve problems involving the calculation and conversion of units of measure, using decimal notation to three decimal places where appropriate • use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to three decimal places • convert between miles and kilometres • recognise that shapes with the same areas can have different perimeters and vice versa • calculate the area of parallelograms and triangles • recognise when it is necessary to use the formulae for area and volume of shapes calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3) and extending to other units, such as mm^3 and km^3.	Using the number line, pupils should use, add and subtract positive and negative integers for measures such as temperature. They should know approximate conversions and be able to tell if an answer is sensible. They should relate the area of rectangles to parallelograms and triangles, and be able to calculate their areas, understanding and using the formula to do this. Pupils could be introduced to other compound units for speed, such as miles per hour, and apply their knowledge in science or other subjects as appropriate.

Geometry: properties of shapes	Geometry: properties of shapes
Pupils should be taught to: - recognise, describe and build simple 3-D shapes, including making nets - compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons - illustrate and name parts of circles, including radius, diameter and circumference - find unknown angles where they meet at a point, are on a straight line, and are vertically opposite.	Pupils should draw shapes and nets accurately, using measuring tools and conventional markings and labels for lines and angles. Pupils should describe the properties of shapes and explain how unknown angles and lengths can be derived from known measurements.

Geometry: position, direction, motion	Geometry: position, direction, motion
Pupils should be taught to: - describe positions on the full coordinate grid (all four quadrants) - draw and translate simple shapes on the coordinate plane, and reflect them in the axes.	Pupils should draw and label a pair of axes in all four quadrants with equal scaling. This extends their knowledge of one quadrant to all four quadrants, including the use of negative numbers. Pupils should draw and label rectangles (including squares), parallelograms and rhombuses, specified by coordinates in the four quadrants, predicting missing coordinates using the properties of shapes.

Data	Data
Pupils should be taught to: • interpret and construct pie charts and line graphs and use these to solve problems • calculate and interpret the mean as an average.	Pupils should connect their work on angles, fractions and percentages to the interpretation of pie charts. Pupils should both encounter and draw graphs relating two variables, arising from their own enquiry and in other subjects. They should connect conversion from kilometres to miles in measure to its graphical representation. Pupils should know when it is appropriate to find the mean of a data set.

Beyond Year 6

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. Beyond year 6 the programme of study is organised into apparently distinct domains, but pupils should build on key stage 2 and connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge in science, geography, computing and other subjects.

Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on.

Working mathematically

Through the mathematics content, pupils should be taught to:

Develop fluency

- consolidate their numerical and mathematical capability and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]
- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics.

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations

- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Beyond Year 6

Beyond Year 6 Programme of Study

Number and place value

Pupils should be taught to:

- understand and use place value for decimals, measures and integers of any size
- order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- recognise and use relationships between operations including inverse operations
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- interpret and compare numbers in standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or zero
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators

Number and place value (continued.)

- use standard units of mass, length, time, money and other measures, including with decimal quantities
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]
- use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \leq b$
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- appreciate the infinite nature of the sets of integers, real and rational numbers.

Algebra

Pupils should be taught to:

- use and interpret algebraic notation, including:
 - ab in place of $a \times b$
 - $3y$ in place of $y + y + y$ and $3 \times y$
 - a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$
 - $\frac{a}{b}$ in place of $a \div b$
 - coefficients written as fractions rather than as decimals
 - brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
 - collecting like terms
 - multiplying a single term over a bracket
 - taking out common factors
 - expanding products of two or more binomials

- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by using graphs
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- work with coordinates in all four quadrants.
- Recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane.

Algebra (continued.)

- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the n th term
- recognise geometric sequences and appreciate other sequences that arise.

Ratio and proportion and rates of change

Pupils should be taught to:

- change freely between related standard units [for example time, length, area, volume/capacity, mass]
- use scale factors, scale diagrams and maps
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part: part or part: whole ratio; express the division of a quantity into two parts as a ratio
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions
- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- use compound units such as speed, unit pricing and density to solve problems.

Geometry and measures

Pupils should be taught to:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)
- calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes
- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- identify properties of, and describe the results of, translations, rotations and reflections applied to given figures
- identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- understand and use the relationship between parallel lines and alternate and corresponding angles

Geometry and measures (continued.)

Pupils should be taught to:

- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs
- use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles
- use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D
- interpret mathematical relationships both algebraically and geometrically.

Probability

Pupils should be taught to:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.

Statistics

Pupils should be taught to:

- describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs.